

2021

## ADVANCED BUSINESS MATHEMATICS — HONOURS

Sixth Paper

(A-32-A)

Full Marks : 50

*The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.*

## Group-A

1. Answer the following questions:

2×5

(a) Find the range of the following function:

$$y = \frac{x^2}{1+x^2}$$

(b) If  $\varphi(x) = 2^{mx+1}$ , then show that  $\varphi(a) \cdot \varphi(b) \cdot \varphi(c) = 4 \cdot \varphi(a + b + c)$ .*Or,*

$$\text{If } f(x) = \frac{x-a}{x} + \frac{x}{x-b}, \text{ show that } f\left(\frac{a+b}{2}\right) = \frac{4ab}{a^2-b^2}.$$

(c) Draw the graph of the function:

$$y = f(x) = x - 1, \quad x > 0$$

$$= -\frac{1}{2}, \quad x = 0$$

$$= x + 1, \quad x < 0$$

(d) Find:  $\lim_{x \rightarrow 1} \frac{e^{\log x} - 1}{e^{x-1} - 1}$ *Or,*

$$\lim_{x \rightarrow 0} \frac{5^x - 4^x}{x}$$

(e) If  $y = \frac{x-2}{x+2}$ , then prove that  $2x \frac{dy}{dx} = 1 - y^2$ .*Or,*Differentiate  $5x^4$  with respect to  $x^2$ .

## Group-B

2. Answer the following questions:

(a) Evaluate:

$$(i) \lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x}$$

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(ii)  $\lim_{x \rightarrow \infty} \frac{6-5x^2}{8x-15x^2}$  3+3

(b) If  $x = y^y y^{\dots \infty}$ , show that  $\frac{dy}{dx} = \frac{y(1-x \log y)}{x^2}$ . 6

(c) If  $2x + 3y = 4$ , find the maximum or minimum value of  $xy$ . 6

**Or,**

Show that the maximum value of  $\left(\frac{1}{x}\right)^x$  is  $e^{1/e}$ . 6

(d) Prove that  $\begin{vmatrix} 2a & a-b-c & 2a \\ 2b & 2b & b-c-a \\ c-a-b & 2c & 2c \end{vmatrix} = (a+b+c)^3$  6

**Or,**

Find the value of  $x$  when  $\begin{vmatrix} x-1 & 1 & 1 \\ 1 & x+1 & -1 \\ -1 & 1 & x+1 \end{vmatrix} = 0$ . 6

(e) Find  $A$  when  $A^{-1} = \begin{bmatrix} 1 & 3 & 2 \\ -3 & -3 & -1 \\ 2 & 1 & 0 \end{bmatrix}$ . 6

**Or,**

Solve the system of linear equations by matrix method: 6

$$2x + 3y + z = 17, \quad x - y + z = 3, \quad 3x + 2y - 2z = 4$$

**Group-C**

3. Answer the following questions:

(a) Evaluate (**any one**): 4×1

(i)  $\int \frac{dx}{\sqrt{x+1} + \sqrt{x+2}}$

(ii)  $\int \frac{x^3}{x^4+x^2-12} dx$

(b) A bag contains 6 white and 4 black balls. 2 balls are drawn at random. Find the probability that 6

(i) they are of same colour,

(ii) they are of different colour.

**Or,**

Two dice are rolled one after the other. Find the probability that the number on the first is smaller than the number of the second. 6

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